



# Super calibrate stochastic exotic option analysis!\*

Calibration, recalibration and co-calibration. These, according to ITO 33, are the keys to unlocking the problem of derivative pricing

**S**ome of you may not be familiar with the terms calibration, recalibration and co-calibration. Some of you may still think that the big challenges facing computational finance today are the following (a) Work hard to find closed form pricing formulas (b) Make sure, when they are not available, that the numerical schemes replacing them have the best accuracy vs. speed ratio (c) Make sure, when developing such a numerical framework, e.g. Monte Carlo or PDE, that you build a generic solving engine

where you can seamlessly integrate any potentially complex derivative payoff.

While “quants in the banks” are left struggling with (a), points (b) and (c) seem to involve enough ingenuity to justify engineering enterprise properly so called; specialized software vendors have indeed been seen recently to advertise just this kind of program. As a matter of fact, we, at ITO 33, were largely responsible for bringing numerical specialization to the fore in quantitative finance, and insisting that it should be dealt with by teams that remain in place a longer time than the average “quant in a bank;” in other words, independent specialized companies just like our own.

This, however, is not the biggest challenge facing the derivative pricing industry today. Building accurate PDE based or Monte Carlo based pricing engines is not everything. And integrating complex payoffs is not the last word either. Both items

may be a prerequisite to a global pricing framework, but they do not define what we refer to as the derivative pricing problem. They relate to numerical analysis or software design, but numerical analysis and software design are not all it takes to solve the problems of modern option science.

## Calibration

The days are long gone when a single (volatility) number was sufficient to price a given derivative instrument. At least, ever since implied volatility was dead and buried. Those were the days when an individual pricing formula, or algorithm, was attached to each individual derivative instrument. People would, for instance, feed a single number in their specific convertible bond-pricing engine and refer to it as the “implied volatility of the convertible bond.” Seldom did they pay heed to the fact that this number was largely dependent on their specific model. “Implied volatility” has no application outside the formula that first gave it its meaning. All extensions of the concept sooner or later end in complete confusion or in complete puzzlement.

“Implied volatility” is the number one must feed into the Black-Scholes pricing formula in order to get the given price of the given European option. There is no “implied volatility” for barrier options, for instance, for there might not even exist a volatility number that you may feed in the corresponding Black-Scholes formula and get the given price of a given barrier.

“Implied volatility” of American options is even more pernicious. Classically, it means the volatility number one should feed into the binomial tree in order to get the quoted option price. Equity options are usually American. This means the concept is widespread and chances are that every time someone speaks of “implied volatility” they have in mind just this number. All is well when there are no dividends and no implied volatility smiles. The number loses all its financial meaning, however, when there are some such. Or at least, it loses all connexions with implied volatility as originally defined in the European case.

The implied volatility of a European option written on a dividend paying stock usually means the volatility number that is input in the modified Black-Scholes formula. And the modified Black-Scholes formula is the formula where the underlying stock variable has been replaced by the forward



price of the stock, i.e. the stock less the present value of future dividends. American options on dividend paying stock, on the other hand, are evaluated in trees or lattices where the explicit process of the underlying is discredited. The volatility number appearing in the algorithm is the diffusion volatility of that process. In order to connect the “implied volatility” of an American option with the “implied volatility” of a European option, one has therefore to be very clear about the discretized stochastic process in the American algorithm. Usually, it is the process of the underlying spot price, not the forward price, and dividends are explicitly modeled as deterministic jumps generating what is known as “jump conditions” for the option price. The “implied volatility” number is then understood as the coefficient measuring the diffusion of the underlying between the jumps, and it can be wildly different from the diffusion coefficient of the otherwise continuous forward price, in case dividends are cash.

The case of smiles is even more perplexing. Equity options usually trade under implied volatility smiles. This means that two options of different strikes and maturity dates do not reflect the same volatility number in the Black-Scholes formula. The question then becomes (disregarding dividends): Should an American and a European option, with otherwise identical terms, be priced with the same “implied volatility” number? Obviously not, for the American option is path-dependent and sensitive to the volatility observed along the path of the underlying all the way to the exercise boundary, while the

implied volatility of the European option is the average of such volatilities up to its maturity. It is not even the case that the American Call and the American Put of same strike and maturity can be priced with the same implied volatility number. As Put-Call parity fails in the American case, the Call and Put become two separate entities with completely separate optimal exercise policies, and whatever relation there might subsist between their two values will itself depend on the smile model.

In other words, one has to solve a full smile problem in order to price the American options, given the prices of the European options. This means one has first to make an assumption on the stochastic structure of the underlying process capable of explaining the European smile (Is it general diffusion, jump-diffusion, stochastic volatility, a mixture?), then calibrate the parameters of that process, and finally price the American option.

The key word here is “calibration.” In practice, it means that the value of a single American option depends not only on the full set of European option prices, but also on the theoretical smile model that one has to choose independently. What adds to the difficulty, in the case of equity options, is that the observed option prices which are constitutive of the “smile” are usually the prices of the American options, not the European. This means you have to solve a (very hard) smile problem, based on calibration to American options and almost certainly complicated by the presence of dividends, before you can even get the price of a European option!

In sum, no proper derivative pricing model (or algorithm, or

engine) is conceivable today, which requires just the entry of a single volatility number and is consequently attached to a single derivative instrument. The engine has to come complete with a full smile model (or models), a calibration procedure, and a universe of other related instruments and prices among which to pick the liquid set to calibrate the model against. This has dramatic consequences on model design and software architecture.

### Recalibration

Calibration is only a technical facility. It is not reality. In itself, it does not pose the real problem of derivative pricing (let alone solve it) and is only part of it. Calibration is just the technical capacity to find a combination of parameters of the given stochastic process such that the theoretical prices of some given derivative instruments match their empirical prices. (Speed of calibration is a real problem, though, and the word “calibration” in our three-tiered maxim matters only to the extent that there is a calibration routine, therefore a calibration overhead, at the core of the engine.)

Reality is recalibration. The real derivative pricing problem begins with recalibration and we must, in a sense, solve a recalibration problem before we consider calibration or even think of the best model to calibrate. As soon as we take stock of what we wrote before, namely that no derivative pricing model is possible today that does not primarily rely on calibration to the market prices of several relevant derivative instruments, we are already commit-

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ted to recalibration, for the market prices of those derivative instruments will certainly change the next day, therefore will pose the question of recalibration.

Take implied volatility. This is the single most significant concept in the entire past history of derivative pricing. (No wonder it is the very concept we are presently striving to evict.) And the reason it is so significant is the particular way it is oriented. The concept of implied volatility flows from reality to the theoretical

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model, not the other way round. Technically, it is the number to plug into the first historical derivative pricing model (the Black-Scholes formula) in order that the model matches empirical reality. This is what gave it its name. On the surface, “implied volatility” just means that we invert the Black-Scholes formula against the option price and imply the corresponding volatility number. But what “implied volatility” really means is that implied volatility will change everyday because the empirical price of the corresponding option will change.

Implied volatility is thus essentially – not accidentally! – a recalibration concept. And to the extent that implied volatility has meant the reality of option pricing in its infancy stage, recalibration will mean its reality in the present, full sense of the term. That there should be no intermediary stage and that we should jump directly from implied volatility to recalibration without even pausing at calibration, is due to the fact that implied volatility concerned a single derivative instrument in each given instance and that calibration was in fact hidden behind a trivial root-finding procedure. We may thus lay it down: The technical generalization of implied volatility is calibration; but its real generalization is recalibration.

Changes of implied volatility precipitated the need of option pricing

models more general than Black-Scholes: the so-called “stochastic volatility models.” The very rich smile literature consequently flourished, and it was dominated by calibration. Indeed, due to what we shall call a “necessary accident” of the smile models that were proposed, everybody’s attention was momentarily diverted from recalibration (when it should have been the next dominant concept after implied volatility).

As stochastic volatility models typically involved more than one parameter and calibration to the time series of implied volatility numbers was out of the question on account of the forward-looking character of derivative pricing, calibration of the model parameters could follow no other route than instant calibration to a collection of different option prices. Happily (or shall we say, unhappily?), stochastic volatility and correlation happen to also produce option smiles and skews, so the temptation was just too great to try to match those.

We think this was the single most unfortunate detour that derivative pricing could ever take.

Indeed the danger was that a technical answer would be mistaken for the real answer and calibration perceived as the ultimate goal. Not mentioning that a metaphysical presupposition was being smuggled in, namely, that the given smile model was the true model and the only question remaining a technical one: how to calibrate its parameters. When implied volatility was all we had, there was no such risk. The simplicity of the concept somehow safeguarded it. Implied volatility was so evidently changing that no one believed it was true. And it was so naturally different from one option to another that no one could forget the shallow empirical concept that it was. (This is the reason why Black-Scholes is so robust by the way.) By contrast, the technical effort involved in the calibration of stochastic volatility models gives a wrong feeling of depth.

Not to say that our favorite smile model should not be calibrated to the present smile! Like we said, this is a technical prerequisite and is just the generalization of the technical meaning of implied volatility. But necessary conditions should not be confused with sufficient ones and the solution of the real derivative-pricing problem, as we come to perceive it now, will consist in smile models that can internalize recalibration.

Traditional stochastic models (Heston, Bates, Pan Duffie Singleton, etc.) fail to internalize recalibration because the only way to recognize the fact that their calibrated parameters will change stochastically over time is to postulate an external stochastic process governing those changes. The chain of models thus never closes off; a given stochastic process is always followed by another, the given model is critiqued by a meta-model and recalibration is always perceived as an accident (or a “dangerous supplement,” to use an expression from Rousseau and Derrida).

Placing recalibration before calibration has deep consequences, and not just on the class of acceptable models or the algorithms that can make them work. It may indeed take nothing short of Derrida’s overturning of the metaphysics of presence and its replacement by the movement of *différance* to provide the philosophical backing of our new science of derivative pricing. Positing recalibration (not calibration) as the founding category and enabling structure of our smile models (and more generally, science) not only implies that the smile models are not true or were not even supposed to be true, but it dismisses truth, or even presence itself as the central mode of thinking. (And presence is here a more general category than truth, as it includes rationalism, empiricism, phenomenology, transcendentalism, pragmatism, in a word, all philosophies whose founding principle, in the last analysis, lies in the order of the logos, whatever its forms: reason,



fact, intuition, condition of possibility, performativity.)

Re-calibration means that we start with the failure of a model rather than with a model, with the reiteration of a procedure (calibration) supposed to bring a given model in coincidence with reality - thereby investing it with presence - rather than with the procedure itself. It means that our philosophy has, from the start, assimilated the paradoxically sounding and far-reaching revision brought up by Derrida's *différance*, namely, that the self-present moment of truth, or the fulfilment of meaning (i.e. perfect calibration), shall forever remain deferred, and that there shall always subsist, in its place, the essentially mute and inanimate and improper, in a word, the essentially different body of the sign (i.e. a material instance of recalibration). The whole category of presence is derivative on the non-present and the radically other; *différance*, as Derrida says, is "older" than *Being*; the ideality of meaning is derivative on repetition; not the other way round. Recalibration, we may say, is an originary repetition.

That recalibration should be our first philosophy and our first enabling structure is not due to the *a posteriori* recognition of the inevitability of recalibration. It is not that the proposed smile models invariably break down in practice and that the parameters that were supposed to remain constant in theory invariably end up stochastic. Rather, one has to recognize the necessity of recalibration as the distinguishing character of derivative pricing, that is to say, as an *a priori* requirement. Derivative pricing models are meant, by definition, to price the derivative instruments for the purposes of trading; and the trading of

those instruments will, by definition, introduce states of the world not previously spanned by the pricing model. Pricing-and-trading European options with the Black-Scholes equation introduces a market for implied volatility and creates new states of the world (of stochastic option prices) not previously accounted for by Black-Scholes. And supplementing the Brownian motion of the underlying with a stochastic process for its volatility (e.g. proposing Heston after Black-Scholes, etc.) will not help because the parameters of the augmented process will in turn become stochastic as soon as the "new" option prices are submitted to trading, etc.

This is the essence of the market, perhaps even its defining characteristic. As such, it should be reflected in the pricing model. Pricing is structurally inseparable from trading; consequently recalibration should be structurally inseparable from the smile model. A key observation about the regime-switching model is that the stochasticization of a regime-switching model is still a regime-switching model. In other words, the meta-model in charge of describing the recalibration of a regime-switching model is identical with the object-model. Another key observation is that we have never assumed, in the build up of our argument, that the calibration set was exhaustive. We may be calibrating to the vanillas and the barrier options today, therefore temporarily settling for an instance of the regime-switching model, but in no way are we hiding from the fact that we would have had to calibrate to an even stranger exotic, non intrinsically replicable by the instruments we were initially calibrating against, had the traded

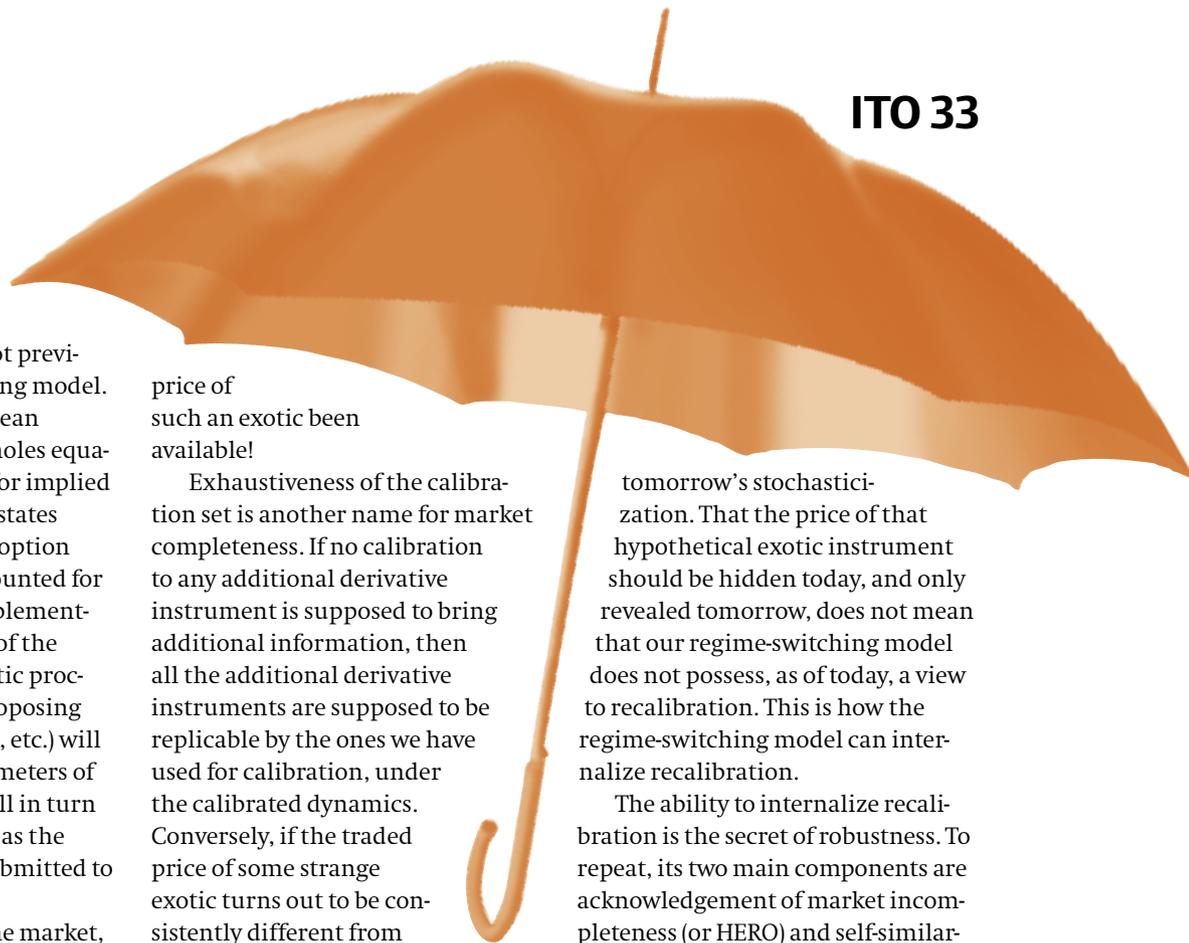
price of such an exotic been available!

Exhaustiveness of the calibration set is another name for market completeness. If no calibration to any additional derivative instrument is supposed to bring additional information, then all the additional derivative instruments are supposed to be replicable by the ones we have used for calibration, under the calibrated dynamics. Conversely, if the traded price of some strange exotic turns out to be consistently different from the price of the dynamic strategy that would replicate it perfectly under the calibrated dynamics, yet this price coexists with the traded prices, as previously calibrated against, of the instruments involved in the replicating strategy, then only a stochasticization of the calibrated dynamics can explain the price of the strange exotic. This circumstance need not present itself in actuality (the exotic need not exist). All we need is its mere possibility and recalibration will be a necessity.

The strange exotic might not exist, let alone trade, today. However, the fact that our regime-switching model will only be numerically, not structurally, different from its stochasticization and the fact that we are computing a HERO in all instances of pricing and hedging today, imply that the very same regime-switching model will be able to price that strange exotic tomorrow with different parameters and that today's HERO may be picking up in advance the result of

tomorrow's stochasticization. That the price of that hypothetical exotic instrument should be hidden today, and only revealed tomorrow, does not mean that our regime-switching model does not possess, as of today, a view to recalibration. This is how the regime-switching model can internalize recalibration.

The ability to internalize recalibration is the secret of robustness. To repeat, its two main components are acknowledgement of market incompleteness (or HERO) and self-similarity of the model through stochastic augmentation. Robustness and recalibration are the real successors of "implied volatility." As such, they are the real solution of the derivative pricing problem, or smile problem. Note that no given instance of the regime-switching model is in itself the solution. Its repetition is. (Or rather, its repeatability, what Derrida would call "iterability.") Individually, the regime-switching model is an inanimate sign, a dead structure whose sole purpose is to get inscribed within the chain of recalibration and indefinite deferral of the moment of presence. No wonder it is nameless ("Nobody"). It is just a written trace; it captures what has always been going on in all other cases of quantitative models that people have been writing, indeed what has always been going on with writing in general. Only it does so more adequately and self-consciously than any other model. In a sense, the quantitative model is always left behind, as a trace, and recalibration always gets





ahead of calibration. The trader is always using the model in the phase where it absents itself from itself and already calls for its recalibration: Always already in *différance*, never in coincidence and self-presence and Being.

This, we said, is the defining characteristic of the derivatives market. Although the derivative instruments are derivative on the underlying (interestingly, they are said to be written on it), and seem for this reason to be secondary with respect to the fully present and fully visible spot market (think, for instance, that no one really questions the pricing of the underlying spot), we wish to invoke the exchange that is typical of deconstruction at this point and wonder whether the spot market, or simply the “market,” may not itself be derivative on the derivative instruments rather than the opposite. For one thing, the replication argument is symmetrical and, instead of looking at some trading strategy in the underlying as the dynamic replication of some complex derivative instrument, one can always look at the derivative instrument as the name of that independent trading strategy. Derivative pricing is just another way of putting a price on the dynamic trading actions that any spot trader is paid to undertake. So could we not, in the end, apply to the underlying what we have written on the instruments that are written on the underlying? Apply to the market at large both the thought and the philosophy of derivative pricing?

Derivatives are not an unnecessary addendum, or supplement, to the market. The way we like to think about them is that they are, in the market, what shapes our thinking about the market (and opens the gate of philosophy). They are the market and have been with us all along. Placing recalibration at the beginning, not just of the derivative pricing model and its specific philosophy, but the beginning of the market and its general philosophy, means that we no longer stop at the particular instance of a model or the particular instance of a derivative instrument, but that we now think of the market as the happening of all this at once! Without him knowing it, the trader dealing with the underlying may very well be “reproducing” a certain derivative and “pricing” this derivative at exactly the significant moment of recalibration. True, this narrative may only be a fiction, but like we said, its mere possibility imposes the necessity of thinking of the market this way.

The metaphysics of presence dictated on us the rational sequence according to which the underlying is first of all firmly given, then a stochastic process is given for it with the corresponding states of the world, then the derivative is given, then its price is computed, then it is traded, then the states of the world are augmented and recalibration is needed. If recalibration is understood as the essence of the market at the end of the chain there is no reason why it shouldn't be also its essence at the beginning, especially when *différance* is making us altogether wary of the whole metaphor of the chain, of the stratum, and of the beginning of the chain. We should really thank the

derivatives for the opportunity they offered us to think recalibration at last. By saying: “In the beginning, there is recalibration,” we are in fact prohibiting the market from ever being framed in a given context, and this is precisely what the market is: an un-representable time-series of changing contexts.

### Co-calibration

Co-calibration achieves in space what recalibration achieves in time. Just as tomorrow's recalibration (and the corresponding stochasticization of the current model) could have been readable today had the price of the corresponding un-replicable exotic been available today and the regime-switching model calibrated to it, different dynamics, affecting a different underlying, could have flown to the present regime-switching model had it been jointly calibrated to the derivatives written on that other underlying. And the ability to internalize co-calibration is just as much a factor of robustness as the ability to internalize recalibration. It is well known that the default risk of an issuing company produces massive negative skew in the option prices written on the equity of the company because of the jump to default. The other components of the option smile and term-structure of implied volatility are explained by stochastic volatility and non-default jumps of the underlying. The probability of default, on the other hand, is measured by the premiums of the credit default swaps that the market writes against the issuer. As the probability of default can be correlated with the equity price and its volatility, the premiums of the CDS will also exhibit sensitivity to the underlying equity price and volatility.

All in all, the equity-to-credit derivative pricing model has to be jointly calibrated to the option price surface and the CDS term-structure of credit premiums. The option skew cannot discriminate between the jump to default and other, possibly substantial, negative jumps of the underlying. Adding the CDS to the calibration set will resolve the ambiguity and result in more robust calibration. Equity default swaps (EDS) may have to be added too and calibrated against, for they act like deep out-of-the-money digital American Puts that help pin down the smile dynamics.

The key observation about the regime-switching model is that it is open to co-calibration as well as recalibration. Indeed, the regimes are abstract states of the world that bear the name of no particular variable. They can be regimes of stochastic volatility, stochastic hazard rate, stochastic recovery rate, stochastic dividends, stochastic interest rate, etc., and bear the names of n-tuples of variables rather than single names. Only the co-calibration procedure and the diversity of the calibration set can write the corresponding names in the corresponding regimes.

Self-similarity across time and space, combined with the recognition of incomplete markets as inscribed in the HERO measure, is the deep reason why the regime-switching representation is potentially more robust, yet more flexible, than any previous proposal. Not mentioning its numerical tractability. We believe it will be the platform for any future derivative pricing framework.

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