The Case for Time Homogeneity

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Abstract
Departure from time homogeneity may be the sign of serious modelling deficiency. We show with three important examples that it is possible to calibrate parsimonious time homogeneous models to complex term structures. Our examples include the volatility smile, the credit spread, and the yield curve.

1 Introduction

We explore a simple yet significant modelling issue in finance. In many situations where market prices display a term structure, it seems natural to resort to some time dependent dynamics if one wishes to calibrate a model to the observed market data. We argue that this is almost always a bad idea, a sign that some important underlying stochastic structure has been missed at the modelling stage.

When a simple model fails to capture some economically meaningful pattern, tweaking a few parameters through time is a dangerous way of getting extra mileage out of an exhausted solution, even if this adjustment yields an excellent calibration. For calibration alone should not measure the quality of a model. Adjusting a few parameters through time for the sake of calibration alone almost always implies crazy future scenarios, which, although not theoretically impossible, look nevertheless often extremely awkward. As a result, tweaked models typically lack robustness and time consistency.

Stability can only be achieved once the salient features of the dynamics of the problem are correctly captured, and this implies in turn a careful description of the underlying state variables. Achieving a good calibration with a time homogenous model is a powerful sign that the stochastic structure of the problem has been correctly formulated. The term structure that we wish to calibrate, like the motion of planets in space, is a complex function of time which may be described by many different time inhomogeneous ad hoc theories. A time homogeneous model in finance resembles the law of gravity in physics. It yields a parsimonious explanation where time does not play a direct role. This feat is achieved at the cost of enlarging the state space, by considering for instance speed and acceleration as additional state variables on top of the position in space.

Increasing the dimension of the state space may prove fatal for the numerical tractability of the model. The brute force solution which consists for instance in replacing every time dependent parameter by a general time homogeneous stochastic process is probably doomed to fail. We search instead for a parsimonious solution, the smallest possible state space on which a time homogeneous dynamics can be written with good calibration properties. It would be foolish to push the analogy with physics too far and claim that we would then have discovered some universal law for finance similar to gravitation. Our goal is merely to seek robustness and stability under the constraint of numerical tractability. The objective of this short essay is to point out that this research agenda deserves a serious consideration.

We show that in many situations, the increase in the complexity of the state space may be limited to the addition of an abstract regime variable which only assumes a small number of states. We investigate three financial environments where the analysis of a term structure is of the essence: the implied volatility smile, the term structure of credit spread, and the yield curve. In each case we obtain encouraging calibration results, and the added difficulty of working with a larger state space is more than offset by the benefits brought by time homogeneity.

TABLE 1. CALIBRATED PARAMETERS OF THE REGIME-SWITCHING MODEL (3 REGIMES)

	Brownian Diffusion	Total volatility
Regime 1	9.57%	11.67%
Regime 2	6.24%	32.23%
Regime 3	2.25%	11.88%

	Jump size	Jump intensity
Regime 1 → Regime 2	−9.07%	0.2370
Regime 2 → Regime 1	62.67%	0.0855
Regime 1 → Regime 3	2.72%	3.3951
Regime 3 → Regime 1	−3.17%	2.9777
Regime 2 → Regime 3	24.63%	1.0944
Regime 3 → Regime 2	−22.66%	0.2040

2 The Implied Volatility Smile

The implied volatility schedule of at the money calls as a function of maturity is a first important example of term structure in finance. It is well known that a simple tweak to the standard time homogeneous Black-Scholes model will do the job: by allowing the volatility parameter to be a function of time, any term structure can be recovered. If one wishes to fit an entire smile schedule across maturity and strike price, this trick can be extended to a so called local volatility by letting the volatility be a function of time and spot price. Anyone who ventured down this avenue knows that the journey ends in a bitter numerical fiasco. The seemingly natural extension is in fact all but natural. It lacks robustness, yields chaotic predictions for future smile patterns, and generates hedges and prices for exotic instruments way out of line with market practices. One could hardly paint a gloomier picture.

The good and somewhat surprising news is that one need not introduce a very sophisticated state space in order to recover time homogeneity. Tables 1, 2 and 3 show that a few regimes with a simple

TABLE 2. QUALITY OF FIT OF A FULL IMPLIED VOLATILITY SURFACE WITH THE REGIME-SWITCHING MODEL. SOURCE: S&P 500 INDEX ON OCTOBER 1995

	Strike										
Maturity (years)	**80**	**85**	**90**	**95**	**100**	**105**	**110**	**115**	**120**	**130**	**140**
	Market	19.00%	16.80%	13.30%	11.30%	10.20%	9.70%				
0.18	**Model**	19.22%	16.38%	13.35%	11.69%	10.38%	10.29%				
	Market	17.70%	15.50%	13.80%	12.50%	10.90%	10.30%	10.00%	11.40%		
0.43	**Model**	17.56%	15.85%	13.97%	12.43%	11.14%	10.08%	10.07%	11.53%		
	Market	17.20%	15.70%	14.40%	13.30%	11.80%	10.40%	10.00%	10.10%		
0.70	**Model**	17.34%	15.90%	14.37%	13.00%	11.85%	10.87%	10.11%	10.20%		
	Market	17.10%	15.90%	14.90%	13.70%	12.70%	11.30%	10.60%	10.30%	10.00%	
0.94	**Model**	17.22%	15.93%	14.60%	13.39%	12.36%	11.47%	10.69%	10.23%	11.04%	
	Market	17.10%	15.90%	15.00%	13.80%	12.80%	11.50%	10.70%	10.30%	9.90%	
1.00	**Model**	17.19%	15.93%	14.65%	13.48%	12.46%	11.60%	10.83%	10.32%	10.86%	
	Market	16.90%	16.00%	15.10%	14.20%	13.30%	12.40%	11.90%	11.30%	10.70%	10.20%
1.50	**Model**	16.99%	15.98%	14.97%	14.03%	13.19%	12.46%	11.80%	11.24%	10.56%	10.89%
	Market	16.90%	16.10%	15.30%	14.50%	13.70%	13.00%	12.60%	11.90%	11.50%	11.10%
2.00	**Model**	16.87%	16.03%	15.20%	14.42%	13.71%	13.07%	12.48%	11.98%	11.17%	10.76%
	Market	16.80%	16.10%	15.50%	14.90%	14.30%	13.70%	13.30%	12.80%	12.40%	12.30%
3.00	**Model**	16.74%	16.12%	15.52%	14.94%	14.40%	13.89%	13.42%	12.99%	12.26%	11.67%
	Market	16.80%	16.20%	15.70%	15.20%	14.80%	14.30%	13.90%	13.50%	13.00%	12.80%
4.00	**Model**	16.68%	16.19%	15.72%	15.26%	14.83%	14.42%	14.03%	13.67%	13.03%	12.48%
	Market	16.80%	16.40%	15.90%	15.40%	15.10%	14.80%	14.40%	14.00%	13.60%	13.20%
5.00	**Model**	16.63%	16.24%	15.85%	15.48%	15.12%	14.78%	14.45%	14.14%	13.58%	13.09%

TABLE 3. QUALITY OF FIT OF THE ONE-TOUCH PRICE STRUCTURE

		One-Touches									
Maturity (years)		**−5%**	**−10%**	**−20%**	**−30%**	**−50%**	**50%**	**30%**	**20%**	**10%**	**5%**
0.175	**Market**	0.58%	−1.16%	−3.70%	−5.27%	−6.38%	−6.14%	−7.91%	−8,35%	−6.30%	−3.70%
	Model	0.60%	−1.16%	−3.70%	−5.27%	−6.38%	−6.17%	−7.90%	−8.36%	−6.28%	−3.67%
1.5	**Market**	7.17%	6.26%	2.51%	−1.67%	−8.25%	−3.13%	−6.92%	−8.22%	−6.91%	−4.09%
	Model	7.17%	6.26%	2.51%	−1.66%	−8.24%	−3.09%	−6.91%	−8.24%	−6.92%	−4.08%
5	**Market**	8.10%	8.70%	7.47%	5.06%	−0.95%	−0.09%	−2.78%	−4.25%	−4.53%	−3.30%
	Model	8.09%	8.69%	7.45%	5.04%	−0.97%	−0.08%	−2.76%	−4.23%	−4.52%	−3.29%

time homogeneous Markov structure are enough to capture the jumps and the stochastic volatility needed to calibrate not only to an entire vanilla option smile schedule, but also to some key liquid exotic instruments such as digital or forward start options[1]. Whereas the vanilla option prices are used for the implied volatility smile calibration, a few liquid exotic instruments help capture the dynamics of the smile. The simple tweak to the Black-Scholes volatility fails so miserably because it cannot capture the smile dynamics, as reflected in the prices of the exotic instruments.

The bad news is that by extending, even a little, the state space, the markets are no longer complete. This means that the perfect delta hedge, the cornerstone of the Black and Scholes analysis, is lost and the heavy machinery of incomplete markets must be brought to bear if one is to derive meaningful dynamic hedging strategies.

3 The Term Structure of Credit Spread

A second example of term structure is the schedule of credit spread of an issuer as a function of maturity. This topic is attracting a lot of attention today with the development of the Equity to Credit paradigm. Insurance instruments such as Credit Default Swaps are becoming liquid for maturities up to five or ten years. In reduced form models, the term structure of credit spreads is often captured by a default intensity parameter which is assumed to be a function of time and spot. One immediately sees the parallel with the local volatility. Tweaking the default intensity does the job and yields simple numerical procedures. But this is achieved at the cost of hiding the stochastic structure of the default process. The term structure contains some key information about this structure which is revealed in a time homogenous framework with a few constant parameters.

Calibrating a slightly more complex model with constant parameters reveals far more on the underlying stochastic nature of the problem than resorting to a seemingly simpler model with fewer parameters which must be tweaked every period. Table 4, 5 and Figure 1 show that a simple model with two or three regimes and a time homogeneous Markov structure captures quite nicely most credit spread patterns, even for relatively long maturities.

TABLE 4. CALIBRATED PARAMETERS OF A TIME-HOMOGENEOUS REGIME-SWITCHING MODEL (TWO REGIMES ONLY)

	Hazard rate
Regime 1	0.15%
Regime 2	7.15%

Jump intensity	
Regime 1 → Regime 2	0.7400
Regime 2 → Regime 1	0.1270

TABLE 5. QUALITY OF FIT OF THE TERM-STRUCTURE OF SPREADS OF CREDIT DEFAULT SWAPS WITH TWO REGIMES IN A REGIME-SWITCHING MODEL. SOURCE: GENERAL MOTORS 30/09/2003

Maturity (years)	Recovery rate	Market	Model
1	0.45	1.08%	1.16%
2	0.45	1.72%	1.78%
3	0.45	2.10%	2.14%
5	0.45	2.65%	2.53%
7	0.45	2.73%	2.72%
10	0.45	2.79%	2.86%
15	0.45	3.00%	2.96%

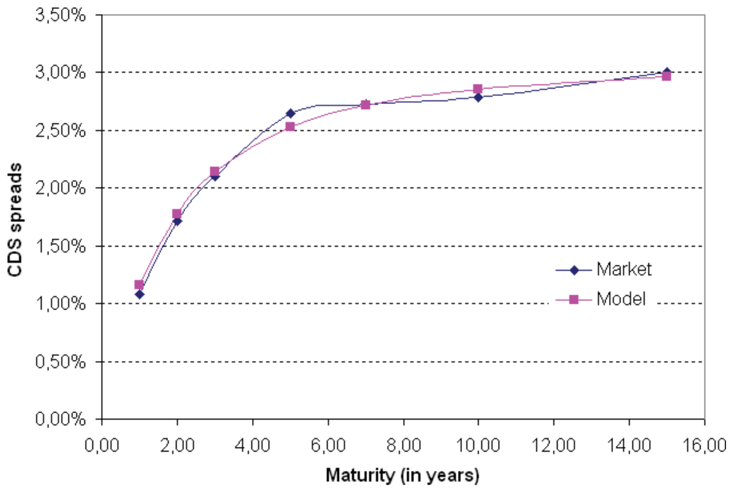

FIGURE 1: Quality of fit the term-structure of spreads of Credit Default Swaps

4 The Yield Curve

A third obvious example of term structure in finance is the yield curve. Two major modelling schools have emerged, which differ in the way they describe the state variable. One school lets the state variable be the short term interest rate while the other one uses the entire yield curve.

The ability to fit a given initial yield curve is a major modelling requirement. For the short term interest rate school, this is achieved by arm twisting the parameters of the short term rate process through time so as to generate the desired yield curve. The second school avoids such painful contortion since the yield curve is viewed as an input, a parameter of the model which need not be calibrated. The main drawback here is that any information on the stochastic structure of the problem which may be contained in the shape of the yield curve is lost.

TABLE 6. CALIBRATED PARAMETERS OF A TIME-HOMOGENEOUS REGIME-SWITCHING MODEL (3 REGIMES) NOVEMBER 1995

	Short rate
Regime 1	5.417%
Regime 2	10.930%
Regime 3	2.626%

	Jump intensity
Regime 1 → Regime 2	0.0402
Regime 2 → Regime 1	0.0783
Regime 1 → Regime 3	0.1903
Regime 3 → Regime 1	0.1005
Regime 2 → Regime 3	0.1574
Regime 3 → Regime 2	0.2615

TABLE 7. QUALITY OF FIT OF THE YIELD CURVE USING THREE REGIMES IN A REGIME-SWITCHING MODEL. SOURCE: US GOVERNMENT ZERO COUPON YIELD CURVES, NOVEMBER 1995

Maturity (years)	Market	Model
0.25	5.410%	5.383%
0.5	5.333%	5.357%
1	5.311%	5.324%
2	5.322%	5.316%
5	5.495%	5.486%
10	5.798%	5.802%

TABLE 8. CALIBRATED PARAMETERS OF A TIME-HOMOGENEOUS REGIME-SWITCHING MODEL (3 REGIMES) OCTOBER 1978

	Short rate
Regime 1	7.388%
Regime 2	0.400%
Regime 3	22.753%

	Jump intensity
Regime 1 → Regime 2	0.6996
Regime 2 → Regime 1	0.5556
Regime 1 → Regime 3	1.5346
Regime 3 → Regime 1	0.4503
Regime 2 → Regime 3	0.7516
Regime 3 → Regime 2	1.6144

For both schools, producing a simple time homogeneous model of the yield curve seems a remote and lost cause. This is a very unfortunate outcome, probably dictated by a more somber agenda: the need to produce quasi closed form pricing solutions, or at least elementary numerical procedures such as one dimensional trees.

It is instructing to realize that a very simple time homogeneous process with no more than three abstract regimes can fit reasonably well almost any yield curve together with the prices of a few interest rates derivatives (see Tables 6, 7, 8, 9 and Figures 2 and 3). Such a model must be solved numerically, but the state variable is so parsimonious that calibration need not be a nightmare.

TABLE 9. QUALITY OF FIT OF THE YIELD CURVE USING THREE REGIMES IN A REGIME-SWITCHING MODEL. SOURCE: US GOVERNMENT ZERO COUPON YIELD CURVES, OCTOBER 1978

Maturity (years)	Market	Model
0.25	8.937%	8.933%
0.5	9.503%	9.513%
1	9.657%	9.640%
2	9.246%	9.261%
5	8.826%	8.819%
10	8.662%	8.664%

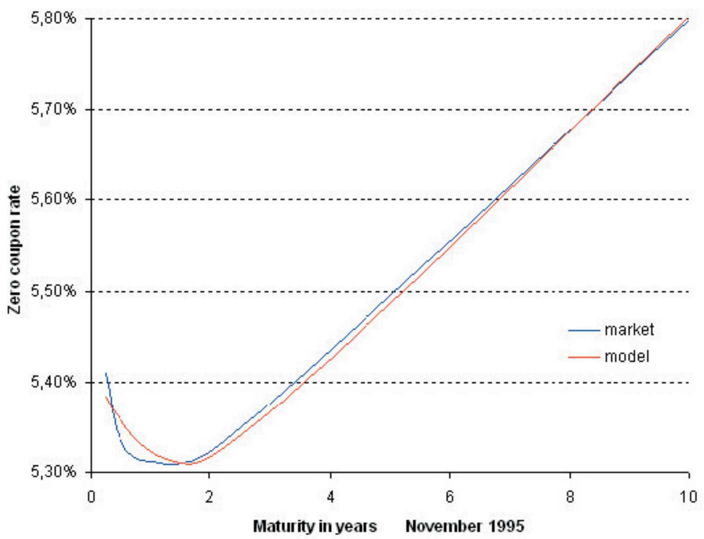

FIGURE 2: Quality of fit of the yield curve using three regimes in a regime-switching model. Source: US Government zero coupon yield curves, November 1995

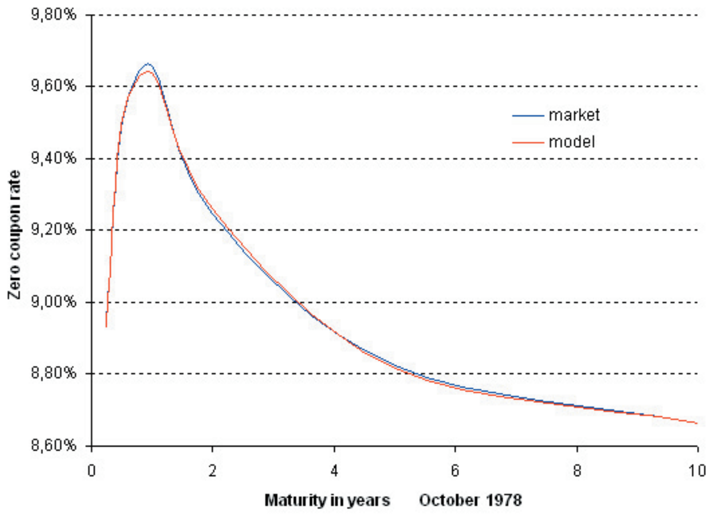

FIGURE 3: Quality of fit of the yield curve using three regimes in a regime-switching model. Source: US Government zero coupon yield curves, October 1978

5 Conclusion

We have made the case for parsimonious time homogeneous models as a powerful way to decipher the stochastic structure underlying a complex collection of market data. In some instances, an event announced for a specific date will destroy the time homogeneity and there are situations where time should indeed be considered as a state variable after all. These cases should be treated as exceptions and not as the rule. We conclude with a simple sanity check for a financial model: any departure from time homogeneity should be the cause of great concerns and should therefore be strongly motivated, lest it is the sign of some serious modelling deficiency.

FOOTNOTE

1. See E. Ayache, P. Henrotte, S. Nassar, and X. Wang. Can anyone solve the smile problem? Wilmott, January 2004.