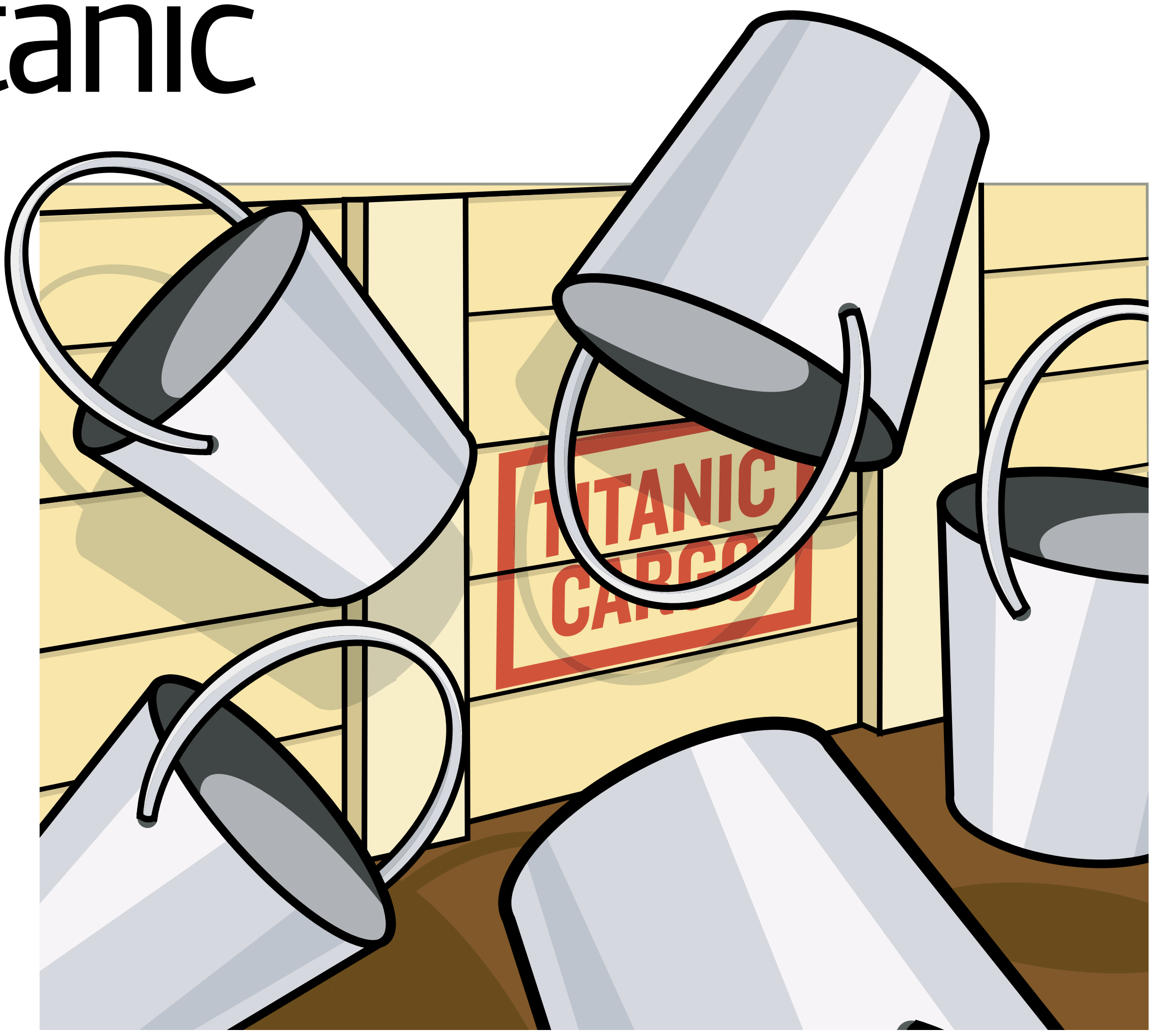
Buckets in the Hold of the Titanic

Out-of-model risk results from the empirical recognition that no model is immune to internally inconsistent re-calibration, Philippe Henrotte writes



Naive bucketing strategies fail to provide consistent hedges but it is not clear either that a multi-layered strategy where a model is refined so as to let its constant parameters become stochastic provides a satisfactory answer from the risk management point of view. We argue that a versatile regime switching model may be a direction of research worthy of investigations.

Out-of-model risk management

It is now well recognized that hedging a derivative instrument is more important than pricing it. Quoting a price for a derivative instrument only makes sense if you stand ready to defend it through an effective hedging strategy. And hedging in turn is closely connected to risk management which worries about the quality of the proposed hedge. There are traditionally two forms of risk management in the realm of derivatives. In-model risk management seeks to derive an optimal hedge from the internal logic of a stochastic model, itself calibrated to fit current market data. In the Black-Scholes setting, in-model risk management corresponds to Delta hedging once the volatility has been implied from the price of an option [5].

Even when theory suggests that this hedge is perfect, traders know by experience that it never takes long for market prices to contradict a model. They re-calibrate constantly their models to current market conditions, and parameters which should be constant are in fact continuously readjusted. We call out-of-model risk management a hedging strategy which takes into account events which are not consistent with the stochastic model used to price the derivative instruments. A standard out-of-model technique controls the portfolio sensitivity with respect to the fixed parameters of a model. We call this technique bucketing by extension from a popular case where the parameters are the values assumed by a term structure at fixed maturity pillars. The Black-Scholes model, for instance, requires continuous re-calibration, which is done by letting the implied volatility assume whatever value is needed to fit the market. If the implied volatility moves constantly, it seems logical to make sure that the value of a hedged portfolio does not change as this abstract coefficient varies. For small variations of the implied volatility, this can be done by keeping a Vega neutral position. If volatility depends on time with fixed values at given pillars so as to fit a term structure of option prices, a bucketing strategy computes a Vega at each pillar.

This out-of-model technique of risk management is not grounded in a theoretical framework and may only offer a false sense of safety if the model parameters follow complex processes with jumps or diffusive volatilities. Hedging the out-of-model risk of a complex derivative instrument by simply controlling its first derivatives with respect to the constant model parameters will not work if the actual time value of the instrument reflects some risk components embedded in the stochastic behavior of these parameters. The naive model with constant parameters will not correctly evaluate the time values of the instrument and continuous out-of-model re-hedging will generate P/L gaps.

The only scientific approach is to embed the moving coefficients in a more complex setting, for instance with a stochastic volatility model, which transforms the inconsistent out-of-model hedge into an optimal in-model one. Alas, one soon realizes that the constant coefficients of the enlarged model also require continuous re-calibration, and consistency can again only be achieved in a new model describing the stochastic process of these parameters [2,3].

We take here a risk management point of view and we focus on the quality of the hedging strategy through time. An additional layer of model is needed anytime an out-of-model event occurs which cannot be hedged correctly by an in-model procedure. Instead, we could have told a story of multi-layered models assuming the more traditional asset pricing point of view. We would have required a new layer of models every time some exotic securities cannot be correctly priced by the last generation of models. The new models would use the market prices of the exotic instruments as a calibration input, only to discover a new generation of instruments which they fail to price correctly. The traditional pricing approach is closely linked to our risk management problem. On the one hand, one needs some additional information in order to calibrate the parameters of an extended model, and it will be provided by the prices of some key exotic securities whose values depend on these parameters. On the other hand, a refined in-model hedging strategy consistent with a more complex stochastic environment requires hedging instruments sensitive to the risk dimensions introduced in the new layer.

The computational complexity grows with each layer of the model, and it is not hard to understand why well trained quants love the affair. The trouble is that it is not clear what this process of multi-layered models achieves in terms of risk management. Are we really better off with a stochastic volatility of stochastic volatility model than with the good old Vega hedging strategy? One could argue that at least on the asset pricing front some clear progress has been achieved if some additional instruments are fitted to the market as each new layer of models is introduced. Each layer depends on a larger set of parameters which presumably offer additional degrees of freedom at the calibration stage. The stochastic volatility model for instance can be calibrated with some barrier options.

The source of this questionable belief can be traced back to a flawed description of risk and uncertainty taught during the very first lectures on financial economics

Even if the calibration abilities of a model say nothing about its out-of-model risk properties, there is a tendency among financial economists to believe that the multi-layered modeling strategy addresses the issue, and that fitting more instruments should result in less out-of-model risk. The source of this questionable belief can be traced back to a flawed description of risk and uncertainty taught during the very first lectures on financial economics. Uncertainty is described by a fixed set of states of the world, and the payoffs of securities are defined as random variables written on these states. No one questions the definition of the states of the world, where they would come from, or the fact that derivatives are not written on abstract states of the world but on the prices of some underlying assets. Here is a good question for an Econ 101 Professor:

"If, as you tell us, a state of the world is defined as a complete description of the relevant economic environment, should not the prices of all assets and commodities be part of the description of a state of the world?"

I guess the answer will be:

"No, the prices will become a function of the state of the world in our next lecture when we shall talk about equilibrium, not the other way round."

If states are fixed and given, then the introduction of new securities help create powerful hedging strategies which help complete the market. If the definition of a state includes the prices of the securities, new securities mean new prices and a larger state space. The new securities yield more powerful hedging strategies, but at the same time the state space becomes more complex. Assuming a fixed abstract state space leads naturally to the flawed belief that uncertainty can be tamed, and that out-of-model risk simply reflects the residual component of the uncertainty that is not yet under control. It implies a series of misconceptions which can be summarized as follows.

- Definition of a good model. A good model eliminates out-of-model risk by proposing a rich enough description of the uncertainty.
- Faith in progress, the weak version. Even if current models are not perfect according to this defi-

nition, each layer reduces out-of-model risk and leads to better models.

● Faith in progress, the strong version. The multi-layered strategy will converge and sometime in the future a model will almost entirely eliminate out-of-model risk [2].

These misconceptions have important consequences for the conduct of risk management. Out-of-model risk is not treated as a fundamental part of modeling, but as a nuisance that can be remedied by enlarging the state space used in the model to better reflect the “real” one, or equivalently by moving to a higher level of model where some constant parameters become stochastic. No matter how this is done as long as more instruments can be fitted in the superior model. They validate the notion that differential calculus and bucketing will efficiently manage the residual out-of-model risk component.

We would like to argue that no real progress is achieved unless one can show that more complex models have superior risk management properties. In other words, let us repeat our initial proposition that hedging is more important than pricing. What matters is not so much the ability to calibrate a very complex model on the prices of a large set of vanilla and exotic instruments at a given time. What really matters is to make sure that a portfolio based on these instruments can be correctly hedged through time. And for this to happen, we need to make sure that continuous re-calibration of a model to market prices does not introduce unmanageable out-of-model risk. The quality of a model should therefore not only be measured by its calibration achievements, but also by the robustness of its hedging strategies through out-of-model re-calibrations. This is a rather complex balancing act where too often robustness is sacrificed on the altar of easy calibration procedures. Bucketing in its most extreme form epitomizes such dangerous modeling choice.

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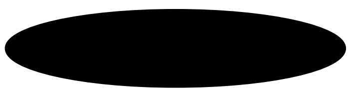

Bucketing

If you think that the process of refining models to better fit the market will ultimately converge and eliminate out-of-model risk, you will also tend to believe that the current state-of-the-art model is not too far from your ultimate goal, and consequently you will treat out-of-model risk as a small perturbation around your current model. The empirical observation that most of times it is possible to re-calibrate your model with only small variations of its coefficients will only reinforce this philosophical stance.

In its most general form, bucketing is an out-of-model risk management technique which computes the sensitivity of a hedging strategy to small variations of the parameters of a model. Its most extreme version, and its name, is derived from the crudest kinds of models where a term structure, say for instance of implied volatility as a function of option maturity, is naively described by assigning values to a series of pillars. Some interpolation technique is then used for maturities falling between the pillars, and each maturity bracket between pillars defines a bucket. Since the parameters of the model are the values at the pillars themselves, it is possible to fit any market data at any degree of accuracy by simply increasing the number of pillars. In our example where volatility is a function of time, in-model risk management is still limited to Delta hedging and the out-of-model risk is described by the Vega assigned to each maturity bucket.

Controlling the Vega of every bucket provides a sense of safety similar to the peace of mind which the water-tight compartment system of the RMS *Titanic* instilled in the officers of the ship. Small and frequent day-to-day price variations are probably well managed by a bucketing strategy which worries about the waves that hit the bow of the *Titanic* as it sails through the North Atlantic at more than twenty knots. It is powerless against the iceberg looming in the dark, where an iceberg is here an out-of-model event significant enough to cause a radical shift in the pricing model needed to price and hedge the derivatives. Its occurrence is likely to be associated with substantial jumps in the prices of the underlying and its derivatives which the naive bucketing strategy will most likely fail to properly address. Just as icebergs are common in the northern seas in winter, disruptive events are not rare in finance. Restructuring, leveraged buyouts, mergers, change of rating, significant news, are but a short list of events which can dramatically affect the life of a company an the behavior of its stock price.

Risk managers have the impossible task of charting the waters for all potential icebergs. They need two sorts of instruments, an early warning system which let them anticipate the significant incoming out-of-model events, and an effective hedging strategy to deal with them. Default may paradoxically not be their greatest worry since any decent equity model would treat bankruptcy as an in-model source of risk.

The market itself may offer a clue as to which iceberg is looming. It is conceivable that the prices of some derivatives incorporate crucial pieces of information on the future of the underlying. The calibration of a versatile enough model may be a powerful tool able to decipher these signals and act as a anti-collision radar.

A model which naively describes a term structure by its values at some pillars will not grasp the dynamics of this term structure and will therefore be unable to deal with exotic instruments linked to this dynamics. These are however precisely the instruments than can best serve as early warning system or hedging tools against potentially dramatic events.

Regime switching

Consider here a simple regime switching stock price model with a Markov transition structure in continuous time between two regimes. In each regime the underlying follows a jump diffusion process with a constant volatility and a constant default intensity. Both the volatility and the default intensity depend on the regime and the underlying may experience a jump whenever a regime shift occurs. This model is versatile enough to jointly fit rich patterns of smile and CDS term structures and to accommodate very different stories. If the two regimes differ only in their volatility or in their default intensity, it can be viewed as either a stochastic volatility model or a stochastic spread model. It can also naturally capture special situations where a shift of pattern occurs in the life of the company. An incoming restructuring of the debt may be described by a regime with lower volatility, lower default intensity and a positive jump when this regime occurs. An incoming leveraged buyout on the other hand will typically lead to a regime with higher volatility, higher default risk, but may also be associated with a positive jump on the stock price.

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Most importantly, the versatility of the regime switching model makes it possible to treat as in-model risk a new source of uncertainty which other models can only view as out-of-model [1, 3]. A stochastic volatility model will not be able for instance to grasp the full implications of an incoming restructuring event and may face the trader with large out-of-model risk as the event unfolds.

This does not mean however that the regime switching model is immune to out-of-model risk. Ultimately only a thorough back test of hedging strategies with inconsistent out-of-model re-calibration can say which model better deal with out-of-model risk [3]. It is unfortunate that most of the academic literature still measures the quality of a model by its ability to price derivatives or to fit the behavior of the underlying under some abstract rational expectation hypothesis, whereas the really interesting feature should be the ability to hedge through time.

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